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REVISITING THE HYPSOMETRIC CURVE AS AN INDICATOR OF FORM AND PROCESS IN TRANSPORT-LIMITED CATCHMENT

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ABSTRACT

Hypsometry has historically been used as an indicator of geomorphic form of catchments and landforms. Yet there has been litle work aimed at relating hypsometry to landform process and scale. This paper uses the SIBERIA catchment evolution model to explore linkages between catchment process and hypsometry. SIBERIA generates results that are qualitatively and quantitatively similar to observed hypsometric curves for physically realistic parameters. However, we show that not only does the hypsometry reflect landscape runoff and erosion process, but it is strongly dependent on channel network and catchment geometry. We show that the width to length ratio of the catchment has a significant influence on the shape of the hypsometric curve, though little on the hypsometric integral. For landforms dominated by fluvial sediment transport, the classic Strahler 'mature' hypsometric curve is only generated for catchments with roughly equal width and length. Narrow catchments show a hypsometric curve more similar to Strahler's 'monadnock' form. For landscapes dominated by diffusive transport, the simulated hypsometric curve is concave-down everywhere, this being consistent with curves reported for some example catchments in France. Because the transition between diffusive dominance to fluvial is scale-dependent, with larger catchments exhibiting greater fluvial dominance, then the hypsometric curve is a scale-dependent descriptor of landforms. Experimental results for simulated landforms from a small-scale rainfall-erosion simulator are reported. It is shown that SIBERIA yields satisfactory fits to the data, confirming its ability to predict the form of the hypsometric curve from a simple model of geomorphic processes. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: hypsometry; channel network; catchment evolution; landform process

INTRODUCTION

The hypsometric curve and the hypsometric integral are non-dimensional measures of the proportion of the catchment above a given elevation. Many researchers have postulated that they are related to catchment form and process (Schumm, 1956; Strahler, 1964). Their primary use has been to characterize the distribution of elevation within a catchment – the study of hypsometry. Some authors have used hypsometry to characterize the distribution of elevations for continental regions so that dependent variables such as regional rainfall and sediment yield may be estimated (Hutchinson and Dowling, 1991; Wyatt, 1993). Other authors have tried to use hypsometry to interpret landform age (Schumm, 1956; Strahler, 1952, 1964). Schumm (1956) examined a waste dump and attributed changes in the hypsometric curve and reductions in the hypsometric integral to increasing catchment age (Figure 1). Strahler (1952, 1964) further asserted that different types of landform have different characteristic shape of their hypsometric curves, dividing landforms into 'young' and 'mature' (Figure 2) with decreasing hypsometric integral – the area under the hypsometric curve – with age. He identified a third landform type referred to as 'monadnock' (Strahler, 1952) which consisted of generally subdued terrain with isolated elevated regions of resistant rock, typical of the New England region of the USA.

There has been little systematic study of the influence of catchment form, process, age and geology of the hypsometric curve and hypsometric integral. This is despite lingering doubts by some researchers about interpretations of the hypsometric curve and its usefulness (Carson and Kirkby, 1972). The main reasons for the lack of systematic study appear to be (1) the laborious nature of obtaining field elevation data in the era before

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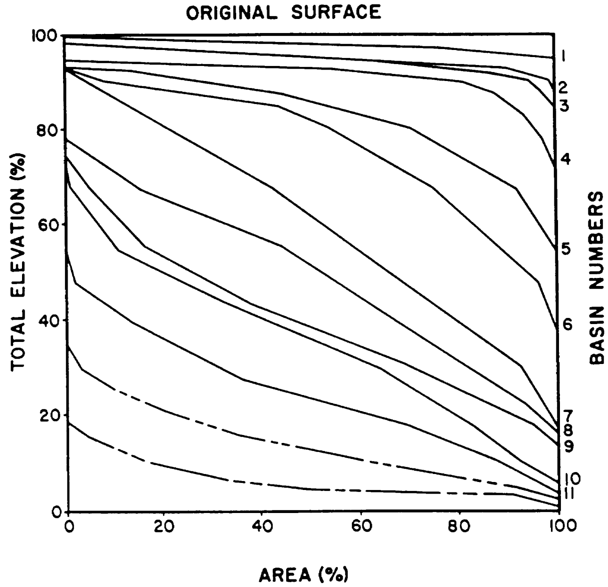

Figure 1. Hypsometric curves reported by Schumm (1956), indicating his speculated trend with age for observed catchments at Perth Amboy

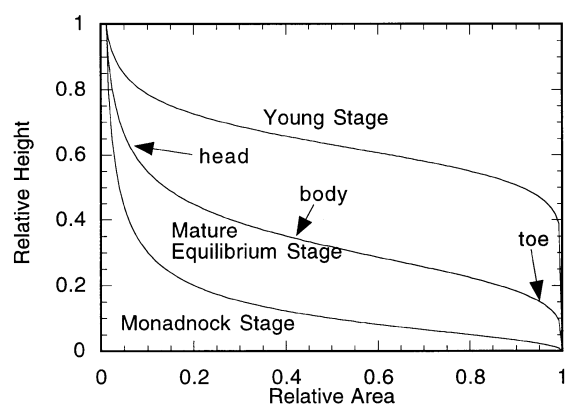

Figure 2. Hypsometric curve classification (after Strahler, 1964). The three regions of the hypsometric curve proposed in this paper are noted on the figure

digital terrain maps, and (2) the difficulties of separating the casual links between process and form in field studies. For instance, age must be inferred by indirect means rather than directly observed in the field, as was, for example, the case for a study of mesa and butte terrain (Strahler, 1952).

With the recent advent of landscape evolution models we are in a position to re-examine the interpretation of hypsometry by looking closely at the causal links between catchment process and landscape form. This paper concentrates on the use of SIBERIA, a catchment evolution model developed by the senior author, to examine this issue, with some of these conclusions being supported by experimental work using a laboratory landscape simulator. SIBERIA is described in detail elsewhere (Willgoose *et al.*, 1991a,b) and the discussion below is confined to the components of the model used in this paper.

Landscapes evolve in response to two competing forces: the tectonic uplift, and the downwasting due to erosion and mass movement processes. This evolution can be formulated as a continuity equation for sediment transport:

$$\frac{\partial z}{\partial t} = U - \nabla q_s \tag{1}$$

where z is the elevation at any point, t is the time, U is the tectonic uplift and q_s is the mass flux of the combined downwasting processes. In general this equation will vary both in space and time, and this equation encompasses all possible transport mechanisms, including both transport- and source-limited fluvial transport. The downwasting model of SIBERIA has two components: (1) a fluvial process dependent on discharge (itself a function of area) and slope, and (2) a diffusion process dependent on slope. The fluvial model used here is based on widely accepted models of transport-limited fluvial transport, while the diffusion model is based on widely used conceptualizations of mass movement mechanisms (e.g. soil creep, rainsplash). We will not address the source-limited case in this paper (e.g. Howard, 1994). The transport model used is:

$$q_s = \beta_1 q^{m_1} S^{n_1} + D\ S \tag{2}$$

$$q = \beta_3 A^{m_3} \tag{3}$$

where q_s is the mass flux per unit width in the direction of the steepest slope (determined from the land surface elevations), q is the discharge per unit width, S is the slope in the steepest downslope direction, A is the area per unit width, D is the diffusivity, and β_1, β_3, m_1, n_1 and m_3 are parameters in the fluvial transport model. When fluvial transport dominates the diffusive process, Willgoose *et al*. (1991c) and Willgoose (1994a) showed that this erosion model leads to a log–log linear relationship between catchment area and slope that can be explained in terms of equilibrium profiles of catchments dependent on erosion, hydrology and geophysics, and which is consistent with observed data reported by Flint (1974), Tarboton *et al*. (1989) and others. Two main equilibrium conditions based on Equations 1–3 have been identified. The first equilibrium condition (Willgoose *et al*., 1991c) was the case where the mean tectonic uplift equals the erosion yield so that mean elevations do not, on average, change with time (i.e. $\partial z/\partial t = 0$), though there may be short-term deviations in response to temporal fluctuations in uplift or climate (Willgoose, 1994b). This condition we describe as steady-state (Hack, 1960). The second equilibrium condition occurs in the absence of tectonic uplift where the catchment asymptotically approaches a Davisian style peneplain. In this case the non-dimensional long-profiles of the catchment approach a constant functional form. This has been labelled declining equilibrium by Willgoose (1994a). For catchments dominated by fluvial transport these equilibria conditions are:

$$\text{steady state:} \quad A^{(m_3 m_1)/n_1} S = \textit{constant} \tag{4a}$$

$$\text{declining equilibrium:} \quad \frac{A^{(m_3 m_1)/n_1} S}{\bar{z}^{1/n_1}} = \textit{constant} \tag{4b}$$

where $\bar{z}$ is the mean elevation of the subcatchment above the peneplain level and S is the slope at the subcatchment outlet. Similarly simple relationships can be derived for catchments dominated by diffusive transport, though for catchments with a combination of these processes acting simultaneously, simple relationships are not possible except for certain parameter values.

This paper will concentrate on the equilibrium hypsometric curves of catchments, arguing that natural landforms approach these asymptotically with time, though, where appropriate, discussion will focus on temporal and non-equilibrium effects. This is felt to be appropriate because recent work has provided evidence that Equation 4 is consistent with observed data and process (Willgoose, 1994a,b) while the observations of Flint (1974) and Tarboton *et al*. (1989) suggest that landforms generally exhibit an approximate log–log linearity between slope and area with some scatter about this trend. Their observations do not assume equilibrium. Even if the catchment is not at equilibrium, a catchment's average area–slope–elevation relationship and planar drainage network (and thus the area draining through every point) allows us to

deterministically reconstruct the elevations of the catchment by integration of Equation 4. Thus the hypsometric curve reflects the area–slope–elevation relationship and drainage pattern.

For both declining equilibria and steady state, the hypsometric curve approaches a long-term stationary curve. Willgoose (1994a) asserted that these stationary curves appeared to be in conflict with Strahler's transition from mature to monadnock as the age of a catchment increases (Strahler, 1964; Figure 2). This paper explores this assertion. In this paper we find it useful to define three parts to the hypsometric curve (Figure 2): the 'toe' of the curve is the downward-concave part of the curve on the right-hand side; the 'head' of the curve is the upward-concave part of the curve in the upper left-hand side; and the 'body' of the curve is the upward-concave region in the centre of the curve between the toe and head.

This paper begins by exploring the effect of catchment planar geometry on the toe of the hypsometric curve; in particular, we examine the effect of the width to length, or aspect, ratio of a catchment. This allows us to generalize published simulation results obtained from hillslope analyses (essentially one-dimensional results) to the more general case of a catchment with a drainage network, while highlighting the limitations of one-dimensional studies. Next we look at the effect of scale and scale-dependent processes on the interpretation of the hypsometric curve. We will conclude with some experimental results from a rainfall-erosion simulator that appear to confirm the conclusions reached in this paper. These analyses will allow us to revisit Strahler's classification scheme.

ONE-DIMENSIONAL LANDFORMS

The simplest and most extensively studied landform is a one-dimensional hillslope, where flow and transport occur parallel to the downslope direction and at a constant rate across the slope. The literature is replete with studies aimed at explicating the relationship between erosion process and long-term hillslope shape (e.g. Gilbert, 1909; Kirkby, 1971; Ahnert, 1987). For the one-dimensional case the hypsometric curve is simply a non-dimensionalization of the hillslope profile. Thus studies of one-dimensional hillslopes have been very important in the conceptualization and interpretation of hypsometric curves, highlighting casual links between the processes, time and shape of the hypsometric curve.

Previous work has established that the typical hillslope profile – concave-down at the divide transitioning to concave-up towards the valley bottom – can be qualitatively matched using a range of different erosion and mass movement processes. These studies concluded that the concave-down region at the hillslope divide requires a strongly slope-dependent process (commonly referred to as a diffusive process) while to obtain the upward-concave regions downstream requires a process that is dependent on both contributing area and slope (commonly referred to as an advective process). The catchment evolution model SIBERIA consists of two such processes and they are sufficient for it to generate realistic hillslopes.

SIBERIA was used to simulate two cases of equilibrium profiles on a one-dimensional hillslope with only fluvial transport. The first case involved an instantaneous uplift of an initially flat hillslope with a fixed downstream elevation. The catchment was allowed to erode with time to develop a one-dimensional hillslope that is in declining equilibrium with the erosion forces acting on it. This 'declining equilibrium' is discussed in detail in Willgoose (1994a). The second case involved applying a continuing uplift with fixed downstream elevation until the erosion on the hillslope equalled the uplift everywhere so that the elevations do not change with time. This continuing uplift is mathematically equivalent to a constant lowering of a knickpoint, studied by other authors (Ahnert, 1987). This 'steady state' is discussed in Willgoose *et al*. (1991c). In both cases the hypsometric curves converge to a fixed curve, the equilibrium profile (Figure 3). Since the area–slope–elevation relation (Equation 4) for each equilibrium is different, the hypsometric curves are slightly different. In both cases the curves are upward-concave everywhere, reflecting the fluvial transport dominance.

In the next series of simulations, the diffusivity of the Fickian diffusion transport term in Equation 2 was gradually increased to yield equilibrium profiles for hillslopes where both fluvial erosion and diffusion were important (Figure 4). As expected, the hillslopes exhibit a downward-concave region near the top of the catchment (where diffusive transport dominates) with the size of the region of downward concavity increasing as the diffusivity increased. For diffusion dominance the difference between the declining and steady-state

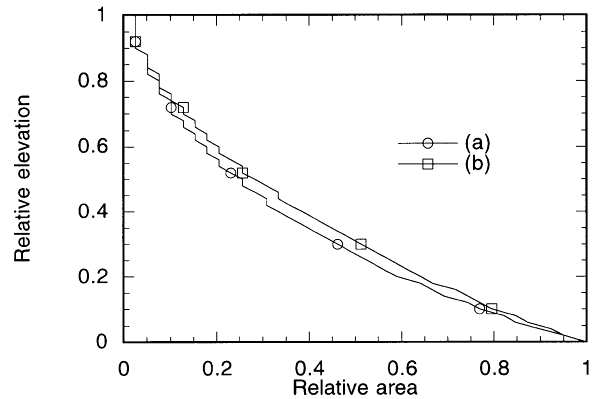

Figure 3. Equilibrium hypsometric curves for (a) declining equilibrium and (b) steady state. Parameters for the simulations: $m_1=2$, $n_1=2$, $m_3=1$

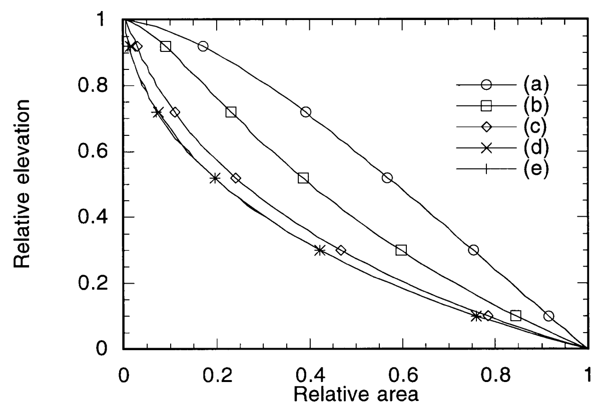

Figure 4. Hypsometric curves for one-dimensional hillslopes for varying diffusivity: $D/\beta_1\beta_3^{m_1}=250\,000$ (a), 2500 (b), 25 (c), 0·25 (d), 0·0 (e). Parameters for the simulations: $m_1=2$, $n_1=2$, $m_3=1$

hypsometric curves and integrals is small. In both cases, the hillslope is concave-downward upstream where diffusion dominates, and concave-upward downstream where the area-dependent fluvial transport dominates.

For these one-dimensional examples at no stage during their evolution do the hypsometric curves show the shape of the 'mature' landform as described by Strahler (1964) (Figure 2). In the absence of diffusion they look like 'monadnock' landforms, while in the presence of diffusion they exhibit a concave-down portion in the top left-hand corner of the curve, observed by Strahler (1952) for a basin near Soisson, France, but not described in his landscape classification scheme. The one-dimensional landforms do not exhibit 'mature' hypsometry at any time.

TWO-DIMENSIONAL LANDFORMS

Hypsometric curves have been used to characterize field catchments (e.g. Schumm, 1956; Strahler, 1964) but numerical studies of two-dimensional (2D) cases equivalent to the studies for one-dimensional (1D) landforms have not been reported. One-dimensional landforms can only provide a crude approximation of the flow divergence, convergence and branching of 2D catchments and, as we will show, can lead to incorrect interpretation of hypsometry.

To explore the effect of two-dimensionality, a number of simulations were carried out using catchments of different aspect ratio. These domains varied from 200×1 nodes to 29×59 nodes with the outlet at the centre of the y dimension so that the aspect ratio ranged from 200 : 1 to 1 : 2. As the aspect ratio decreases and the

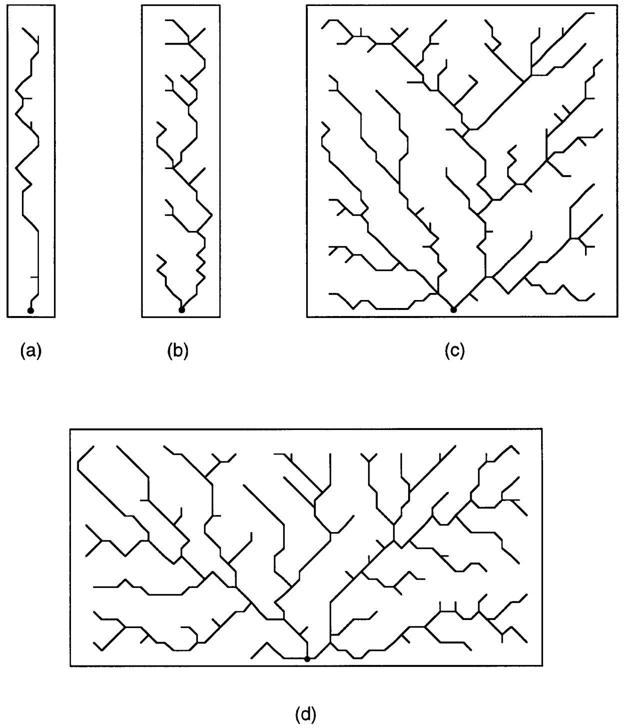

Figure 5. Main drainage paths for the catchment with different aspect ratios. Note that the illustrated drainage paths are not channels as simulated by SIBERIA, but simply the major drainage paths of the landform highlighted to show the two-dimensional nature of the drainage for approximately the same drainage density: (a) 39×5 nodes, aspect ratio 7.8:1; (b) 39×9 nodes, aspect ratio 4·3:1; (c) 39×39 nodes, aspect ratio 1:1; (d) 29×59 nodes, aspect ratio 0·5:1

catchment widens, the catchment has an increasing ability to form two-dimensional drainage paths (Figure 5). The 200×1 catchment simulates a one-dimensional slope. The 29×59 catchment simulates the case of least lateral constraint on the network shape by boundary conditions. These simulations use fluvial parameters $m_1=n_1=2$, $m_3=1$ and zero diffusivity, but the conclusions below are applicable to other fluvial transport parameter values. The catchments have been subject to an initial pulse uplift of a flat plane with small initial elevation perturbations, and then allowed to decline with time to the declining equilibrium. The case of steady state with continuing uplift was also examined. The equilibrium hypsometric curves are presented for varying aspect ratio (Figure 6).

As the aspect ratio decreases the lateral constraint on the drainage network diminishes and the main drainage pattern becomes more highly branched (Figure 5). Accompanying this increased branching is the development of a higher toe (Figure 6). These conclusions are true for both declining equilibrium or steady state. The ordinates of the hypsometric curve for steady state are higher than for declining equilibrium, as for the 1D catchments, but broadly similar.

In this two-dimensional case the evolution of the hypsometry initially follows the Strahler (1964) conceptualization, progressing from ‘young’ to ‘mature’ with increasing age. Different initial conditions yield identical ‘mature’ hypsometry (e.g. Willgoose *et al*., 1989) even though the exact drainage pattern is different for each case. As suggested by Strahler (1964), the landforms remain in this state forever, required by the area–slope results of Equation 4, and the hypsometric integral is in the range 0·4–0·6 observed by Strahler.

The addition of diffusion to the 2D catchments has similar qualitative consequences as for the 1D catchments but because of the different distribution of drainage area in the catchment a greater proportion of the 2D catchment is dominated by diffusion for a given value of $D/\beta_1\beta_3^{m_1}$. The 2D catchment has a greater proportion of its total area as shorter hillslopes than the 1D catchment. Thus the effects of diffusion are apparent at lower diffusivities in the 2D catchment than in the 1D catchment.

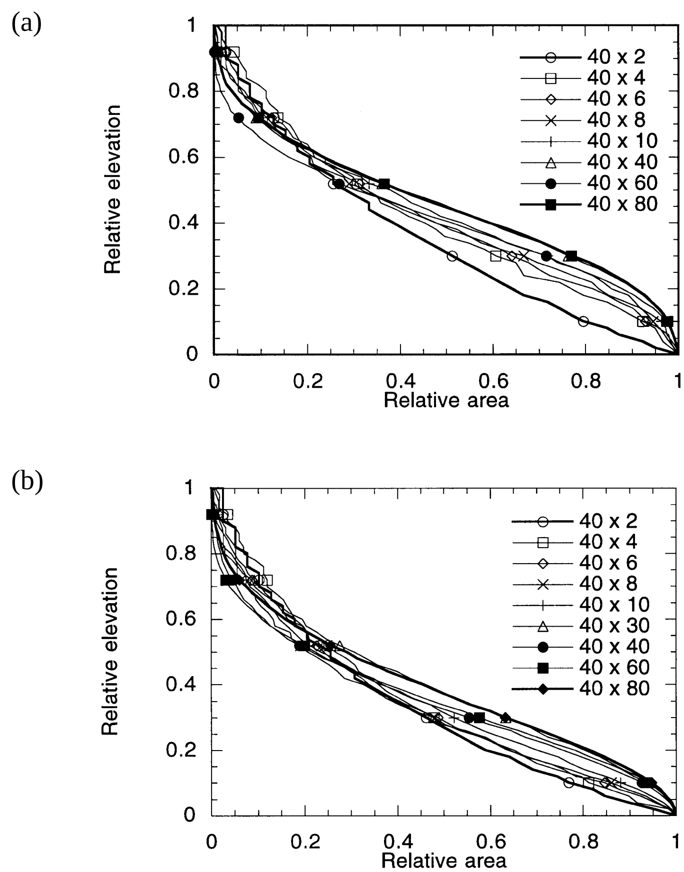

Figure 6. Hypsometric curves for the catchments with different aspect ratios: (a) steady state; (b) declining equilibrium. Parameters for the simulation: $m_1=2$, $n_1=2$, $m_3=1$

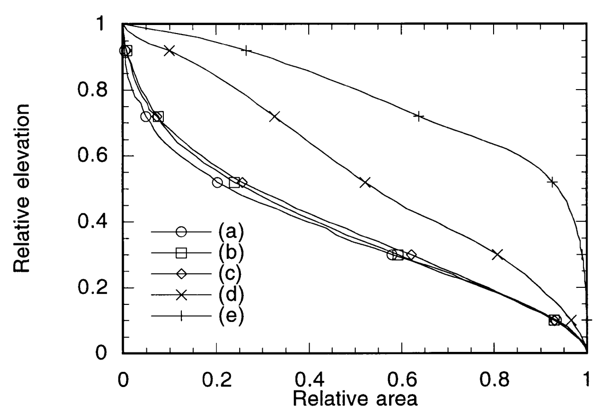

Figure 7. Hypsometric curves for the square catchments (40×40) for varying rates of diffusivity: $D/\beta_1\beta_3^{m_1}=0{\cdot}0$ (a), 0·25 (b), 25 (c), 2500 (d), 250 000 (e). Parameters for the simulation: $m_1=2$, $n_1=2$, $m_3=1$

The effect of diffusion is first observed at the head of the hypsometric curve at steady state in the higher upstream areas, and the ordinates of the curve increase with increasing diffusion (Figure 7). For the highest level of diffusion plotted, almost all the catchment is diffusion dominated and a much higher hypsometric curve

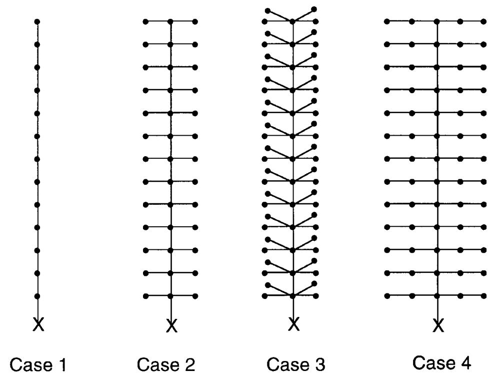

Figure 8. Simple drainage path geometries for explaining the presence of a toe in hypsometric curves

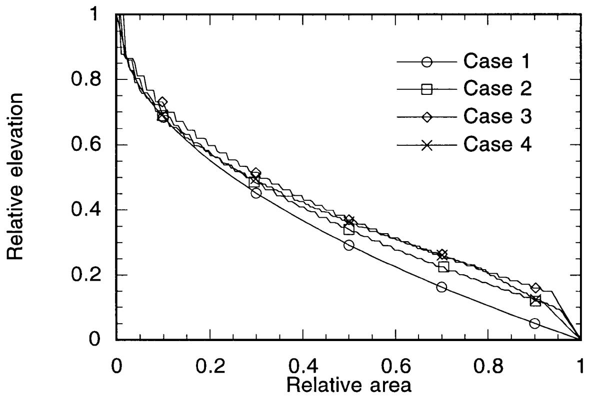

Figure 9. Hypsometric curves for the simple drainage path geometries

is observed than for 1D. This is because of the convergence of the catchment in the downstream direction so that a larger proportion of the catchment is at a higher elevation.

EXPLANATION OF THE TWO-DIMENSIONAL HYPSOMETRIC CURVE

The creation of the toe in the hypsometric curve can be explained by examining some simple catchment geometries (Figure 8) where the elevation properties are given by Equation 4a. These catchment geometries involve a one-dimensional main drainage path with varying degrees of lateral inflow. While not typical of observed catchments, these simple catchments illustrate the causal link between drainage network geometry and hypsometry. Each node in Figure 8 has unit area and slope given by:

$$A^{0{\cdot}5}S=1 \tag{5}$$

where the scaling exponent on area, 0·5, is in the range observed in the field. Case 1, the one-dimensional catchment, has a hypsometric curve without a toe (Figure 9), as for the 1D examples in Figure 3. Cases 2, 3 and 4 all exhibit a toe in the hypsometric curve to varying degrees. Case 2 illustrates the reason for the toe. The long section of the lateral drainage and the main drainage path are identical except that the lateral contributing areas are one unit higher than the main channel, or 0·13 units using the vertical scale of Figure 9. The segment A–B in Figure 9 corresponds to the elevation contribution of the main drainage path that has elevations between 0 and 1 unit. Point B is the elevation above which the lateral areas contribute to the hypsometric curve, so segment B–C corresponds to the combined elevation contribution of the main drainage path and the lateral areas. Cases 3 and 4 show that while the form of the lateral drainage affects the toe, the remainder of the hypsometric curve is relatively unaffected.

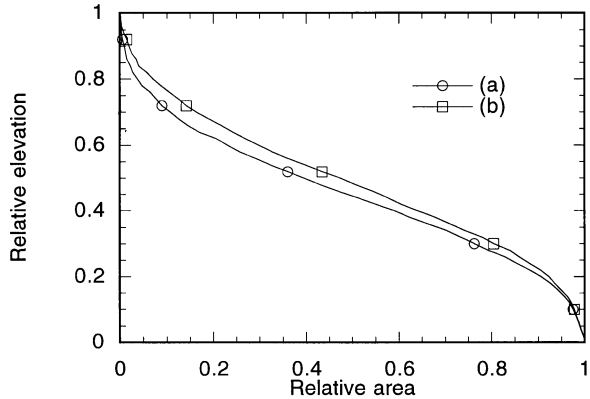

Figure 10. Effect of drainage pattern of the hypsometric curve as a result of a change in process with invariant area–slope relationship: (a) $m_1=2$, $n_1=2$, (b) $m_1=1{\cdot}75$, $n_1=1{\cdot}5$

Thus the toe of the hypsometric curve corresponds to the increasing elevation contributions of the hillslopes in the lower elevation, downstream parts of the catchment. The downward concavity of the toe results from the increasing contribution (as a proportion of the area below that elevation) to the elevation mass of short and steep lateral inflows (whether they be channels or hillslopes) as the elevation increases. Figure 6 suggests that as the catchment has greater freedom to branch and form networks (with the short and steep downstream lateral contributions downstream) then the size of the toe increases. The exact geometry of the lateral inflow also changes the shape of the toe (Figure 9), so that (1) the shape of the toe reflects the form of lateral contribution and (2) the height of the toe reflects the degree of branching within the catchment.

PROCESS AND DRAINAGE NETWORK FORM

Moglen and Bras (1994) found, using the cumulative area diagram, that those parts of the catchment dominated by fluvial sediment transport have different drainage network properties from those portions dominated by diffusive transport. The equifinality of process (Equation 4) means that two catchments may have different fluvial transport processes (i.e. different m_1, m_3 or n_1) but have the same long-section (i.e. the same $(m_1 m_3–1)/n_1$). The question remains whether subtle statistical changes in the drainage network occur with small changes in fluvial transport parameters but not in the area–slope relation, and can they be distinguished with the hypsometric curve?

Willgoose (1994b) speculated that the hypsometric curve might be a sensitive indicator of drainage network form. If a catchment complies with the area–slope–elevation results and the drainage network is known, then the drainage areas draining through every point in the catchment can be determined, and thus the slopes using the area–slope relationship. From these data alone the elevations of the catchment can be reconstructed so that if catchments have identical area–slope–elevation relations then the only way they can have different hypsometric curves is if their drainage networks are different. In this way we may be able to overcome some of the problems of insensitivity in existing network statistics (e.g. Strahler laws).

To test the hypothesis that catchment drainage networks could be distinguished, two simulations were performed for a catchment in steady state, where the area–slope relations were the same but the parameters m_1 and n_1 were different. Figure 10 shows that the hypsometric curves are slightly different, yet the drainage networks appear to the eye indistinguishable. We conclude that (1) drainage networks are affected by process, even if the area–slope relation is unchanged, and (2) the hypsometric curve can distinguish that these drainage networks are different.

EXPERIMENTAL VALIDATION

The predictions of the effect of catchment geometry on the hypsometric curve have been tested by the use of a rainfall-erosion simulator, which simulates the evolution of landforms in the laboratory. This work is part of a

larger programme to quantitatively test the predictions of SIBERIA. The entirety of this experimental programme is reported elsewhere (Hancock, 1997). We will discuss some of the hypsometric curve results.

Rainfall-erosion simulators have been used previously to explore qualitative aspects of catchment geomorphology (e.g. Schumm *et al*., 1987) but to date none have been used to provide quantitative validation of catchment geomorphology models. The University of Newcastle simulator has been specifically designed to test quantitative predictions of the evolution of catchment form. It consists off an erosion tray 1500 mm square with a runoff outlet that is positioned on one of its sides. Rainfall is provided by mist sprays. This rainfall is approximately spatially uniform and of a small enough drop size (diameter 0·05–0·15 mm) that diffusive rainsplash transport is small relative to the advective overland fluvial erosion so that the landscape is dominated entirely by fluvial transport. The eroded experimental material was a fly-ash mixture with a grading in the range 50–90 μm. Runoff generation is Hortonian saturation excess.

Two experiments have been carried out of relevance to this paper. The first experiment created a hillslope of dimension 1000 mm×100 mm, approximating the one-dimensional slopes discussed above. The second experiment used the entire 1500 mm×1500 mm simulator with the outlet in the centre of one of the sides, corresponding to the square (39×39 nodes) two-dimensional case discussed above. Both experiments were allowed to erode from their initial, slightly sloping, planar conditions until the hypsometric curve reached declining equilibrium. For the first experiment, elevations were measured directly at 100 points at a spacing of 50 mm longitudinally and 20 mm laterally. In the second experiment 400 elevations were measured on a 20×20 grid using stereo photogrammetry. Hypsometric curves were then derived directly from these elevations.

These experimentally derived hypsometric curves were then compared with the predictions of SIBERIA for the same catchment geometries. To obtain the parameters for the runoff and erosion models, the equilibrium profiles from SIBERIA were fitted to the one-dimensional hillslope. Parameters $n_1=2{\cdot}1$ and $m_3=1$ yielded a satisfactory fit for $m_1=1{\cdot}62$ (Figure 11).

These fitted parameters were then used in SIBERIA to simulate the landform for the 1500 mm square experiment. The predicted and measured hypsometric curves are presented in Figure 12. The overall fit is satisfactory. The key feature of both the experiment and simulation is the good fit to the shape and height of the toe of the hypsometric curve. The slight deviation of the model in the head is largely due to numerical diffusion in SIBERIA.

The experiments confirm two things. Firstly, that the toe results from the two-dimensional nature of the drainage paths since it is only visible in the two-dimensional experiment. Secondly, the match between the experiments and simulations confirms that SIBERIA satisfactorily matches the two-dimensional nature of the drainage network in so far as it influences the hypsometric curve, strengthening the validity of the preceding numerical studies.

DISCUSSION

We have examined several aspects of catchment process and its effect on the hypsometric curve. These results have important implications for the interpretation of hypsometry.

Hypsometric curves are clearly dependent on the topological shape of the catchment. An elongated catchment has a different hypsometric curve from a wider catchment even if the processes, geology and climate acting in each of the catchments are identical. Moreover, an interpretation based on the Strahler idea of young, mature and monadnock catchments may give an erroneous idea of the relative ages of catchments. What might be interpreted under the Strahler scheme as differences in age may be simply differing catchment geometry. In particular, it is tempting under the Strahler scheme to intepret a monadnock catchment as being solely a result of systematic variation of erodibility within the catchment with rock outcrops at catchment divides. Elongated catchments will also exhibit this profile.

While all the elongated catchments studied in this paper were straight, the derivations of the results for the synthetic networks Cases 1–4 do not assume physical straightness since these were topological representations of networks. Thus elongated catchments that have tortuous planar drainage paths are also covered by the discussions here. The key issue is the amount of network branching that occurs. A catchment with a high

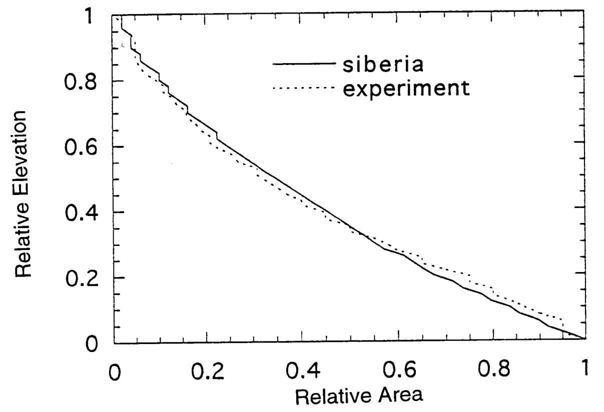

Figure 11. One-dimensional rainfall-erosion simulator results and the fitted SIBERIA simulation

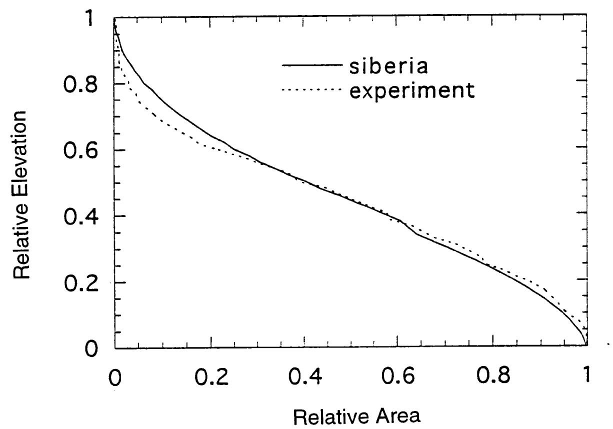

Figure 12. Two-dimensional rainfall-erosion simulator results and the predicted SIBERIA simulation

bifurcation ratio (Strahler, 1964) will exhibit a more two-dimensional hypsometric curve than one with a lower bifurcation ratio.

The reason for the dependence of the hypsometric curve on the catchment shape is because the hypsometric curve is a sensitive indicator of the planar form of the drainage network. How sensitive it is was exemplified by the analysis of the two catchments with identical catchment boundaries and area–slope relation but different sediment transport parameters. The effect on the drainage network was too subtle to detect by eye, yet the hypsometric curve indicated a difference.

The results with simultaneous fluvial and diffusive transport also clearly show that the hypsometric curve is scale dependent and confirm the results obtained by others for 1D catchments work (Kirkby, 1971; Ahnert, 1987). This is consistent with the scale dependence observed in the area–slope relationship (Willgoose *et al.*, 1991c) from which the hypsometric curve is derived. Hypsometric curves derived from a given landform will be differing depending on the area of the catchment adopted for study. A smaller catchment will have a higher proportion of its area dominated by diffusion than a larger catchment, so that the hypsometric curve for the smaller catchment will look like higher values of $D/\beta_1\beta_3^{m_1}$ (Figures 4 and 7). This effect can be quantified with a scaling analysis of the relative size of the difusion and fluvial transport processes in Equation 2 (Willgoose *et al.*, 1991d):

$$D/F = \frac{DS}{\beta_1 q^{m_1} S^{n_1}} = \frac{D}{\beta_1 \beta_3^{m_1}} \langle A\rangle^{-m_1 m_3} \langle S\rangle^{1-n_1} \frac{S'^{1-n_1}}{A'^{m_1 m_3}} \tag{6}$$

where the slope is expressed as $S=\langle S\rangle S'$, where $\langle S\rangle$ is the slope scale and S' is the scaled non-dimensional slope, and the area is expressed as $A=\langle A\rangle A'$, where $\langle A\rangle$ is the area scale and A' is the scaled non-dimensional area. This equation indicates that for a landform with given parameters D, β_1 and β_3, and typical values for m_1 and m_3 (i.e. $m_1 \approx 2$ and $m_3 \approx 1$), the relative importance of the diffusion process increases (i.e. D/F increases) with decreasing catchment area. If the catchment complies with the area–slope relationship of Equation 4a, then:

$$D/F = \frac{DK}{\beta\ \beta^{m_1}} \langle A\rangle^{(1-m_1 m_3 - n_1/n_1)} \quad A'^{(1-m_1 m_3 - n_1/n_1)} \tag{7}$$

where K is the constant on the right-hand side of Equation 4a. For the typical parameters listed above this indicates that the ratio of diffusion to fluvial transport scales with catchment area is:

$$D/F \propto \langle A\rangle^{-1 \cdot 5} \tag{8}$$

Using Equation 8, Table I indicates the relative area of catchments that correspond to the ratios of $D/\beta_1\beta_3^{m_1}$ in Figures 4 and 7 and quantifies previous qualitative conclusions in this area.

With regard to the existence or otherwise of the toe of hypsometric curve, Strahler (1952) examined the hypsometry of a range of catchments from around the world producing a general functional form that he felt could be fitted to the hypsometric curves of most landforms:

$$y = \left[\frac{d-x}{x}\ \frac{a}{d-a}\right]^z \tag{9}$$

where a, d and z are fitting parameters. This equation has been used as a basis for a classification scheme for landscapes. This equation will always have a toe, irrespective of parameters, which is not seen in the 1D catchment examined here. Nor will the equation fit the curves for diffusion-dominated catchments.

In conclusion, we assert that the landform classification scheme proposed by Strahler (1952, 1964), while a useful first step towards a general framework for the study of landform hypsometry, fails to account for significant effects that may be important in the field, as a result of, for instance, the scale dependence of diffusive and fluvial transport. In addition, it fails to recognize that the same landform when viewed at different scales will yield different hypsometric curves. Finally, it fails to account for catchment shape, and in fact is misleading in asserting that the monadnock profile results only from a systematic variation of erodibilities with rock outcrops at catchment divides.

Table I. Catchment area versus relative dominance of diffusion and fluvial transport

$D/\beta_1\beta_3^{m_1}$	Area*
0	∞
0·25	10 000
25	464
2500	22
250 000	1

* All areas are relative to the area (assumed to be unity) of a catchment with $D/\beta_1\beta_3^{m_1}$ =250 000

On the other hand, for mathematical modellers this sensitivity to catchment drainage network can be used as a statistic for differentiating catchments, or calibrating models (e.g. Moglen and Bras, 1994). Used in this way we have shown that SIBERIA does a good job of matching the hypsometry of results obtained from a laboratory experiment. These results provide confirmation of the validity of the SIBERIA model formulation and constitutive equations and its ability to provide an understanding of catchment evolution and linkages between the landform and its hydrology.

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